INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2014-15
Statistics - II, Semesteral Examination, May 5, 2015
Answer all questions

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from Exp(λ) with density $f(x|\lambda) = \lambda \exp(-\lambda x), x > 0$, where $\lambda > 0$ is unknown. Consider testing at level α

$$H_0 : \lambda \leq 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of UMP test are satisfied here.
(b) Derive the UMP test of level α.
(c) Consider the test which rejects H_0 whenever $2X_1 < C$ where C is $\chi^2_{1,\alpha}$, the α quantile of χ^2_1. Show that this is a level α test but it is not UMP. **[11]**

2. Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, respectively, from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, where μ_1, μ_2 and σ^2 are unknown.
(a) Find the MLE of (μ_1, μ_2, σ^2).
(b) Derive the generalized likelihood ratio test for testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$. **[12]**

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from Poisson(λ), where $\lambda > 0$ is unknown.
(a) Find the Fisher Information $I_1(\lambda)$ contained in X_1.
(b) Find an estimator $T_n = T_n(X_1, \ldots, X_n)$ such that

$$\sqrt{n}(T_n - \lambda) \longrightarrow N(0, 1/I_1(\lambda)).$$

(c) Find a transform $g(T_n)$ which is asymptotically normal and also has asymptotic variance free of λ. **[12]**

4. The weekly number of fires, X, in a town has the $Poisson(\theta)$ distribution. The numbers of fires observed for five weekly periods were 0, 1, 1, 0, 0. Assume that the prior distribution on θ is

$$\pi(\theta) \propto \theta \exp(-10\theta) I_{(0,\infty)}(\theta).$$

(a) Derive the posterior distribution θ given the data.
(b) Find the highest posterior density estimate of θ. Compare this with the maximum likelihood estimate of θ.
(c) Find the posterior mean and posterior standard deviation of θ. **[15]**